The Probability of Winning at Solitaire:
A Preliminary Description of a State-Transition Approach

Robert S. Todd, Ph.D. And Tim Dorr, Ph.D.
University of Bridgeport, Bridgeport, CT

Abstract:

This paper is the very beginning of a proposal of an approach that may offer a method for solving a currently unsolved problem – calculating the probability of winning at Solitaire. We describe the game as a series of state transitions, from an initial state to a winning state. The state variables and allowable transitions are described, and a list of “next steps” is given. It is hoped that this approach may prove fruitful.

The probability of winning at Solitaire is an unsolved problem. We do not present a solution, but we propose an approach that may be fruitful. (It’s also necessary to confess that we began this project just two weeks ago and have given the literature only a casual perusal; this may not be original.)

The variation of Solitaire that we discuss is the “Draw 3” version of the Solitaire game that’s provided on all Windows computers. The moves allowed in the Windows game are the moves that we will allow. As in the computer game, an unlimited number passes through the deck is allowed. Unlike the computer game, we have complete knowledge of the location of every card.

Computer simulations have demonstrated that the probability of winning, given complete knowledge of all the cards’ positions, is between 84% and 91%.

And since there are exactly:
80,658,175,170,943,878,571,660,636,856,403,766,975,289,505,440,883,277,824,000,000,000,000
permutations of the deck, our approach will be to find the probability of losing.

Discussion:

When the deck is dealt and play is ready to be begun, 15 of the 52 cards are visible: the top seven cards on the “piles” and every third card in the “turn-over deck,” or simply the “deck.” We know where all the other cards are and none are visible. Only visible cards are eligible to be played.

The configuration of the cards at any point in the game is defined by three variables:

POSITION, VALUE, AND STATUS

Position:

The position variable (“Pos”) has 133 values denoted by X*i*,*j*, Y*i*, and Z_i, where “X” refers to the seven stacks at the bottom of the screen where most of the play occurs, “Y” refers to the “turn-over deck” in the upper left part of the screen, and “Z” refers to the four stacks at the top right where, in a winning game, all the cards wind up, sorted by suit and ordered from Ace to King.

The X subscripts have the following meaning. There are seven stacks of cards. The subscript “*i*” identifies the stack position, from left to right. The “*j*” subscript identifies where in the stack a card resides, starting from the bottom. As play progresses, there can be as many as 18 cards on one stack and as few as none.

The Y subscripts start with Y_1 at the top of the deck and increase to Y_{24}, the last card to be turned over.

The Z subscripts start on the left and increase to the right.

Value:

The value variable (“Val”) refers to what occupies a particular position. It is usually the card value and its suit, e.g., AS, KS, QS, …, 4C, 3C, 2C for an unshuffled deck.

As positions are emptied, the value of the position becomes Φ. The notation for a general card is F$, denoting some face value and some suit. (The “$” is used to distinguish a general suit from a spade.)

Status:

The status variable (“Stat”) is either “visible” (V) or “not visible” ($\overline{V}$).

To illustrate the use of this notation, the end point of a winning game is denoted as $Zi = K\$\ \forall i$, $Xi,j = \Phi\ \forall i,j$, and $Yi = \Phi\ \forall i$.

State Transitions:

All allowable transitions from the current state to another state need to be listed. One example of an allowable transition is this: Suppose the AS is the visible card in the left-most stack when the game begins, and you wish to put that ace “up.” The transition is denoted by ($X_{1,1}$, AS, V) → (Z_1, Φ, V). This means “if ($X_{1,1}$, AS, V) and (Z_1, Φ, V), then the transition is allowed.”

If this transition is made, the post-transition state of $X_{1,1}$ becomes ($X_{1,1}$, Φ, V) and the post-transition state of Z_1 becomes (Z_1, AS, V).

Table 1 (below) gives a partial list of allowable transitions, without any attempt to record the state changes associated with them.

Allowed Transitions

1	(X*i*,*j*, A$, V) → (Z*i*, Φ, V)
2	(Y*i*, A$, V) → (Z*i*, Φ, V)
3	(X*i*,*j*, KS, V) → (Z*i*, QS, V)
4	(Y_i, KS, V) → (Z_i, QS, V)
5	(X*i*,*j*, KS, V) → ($X_{i,1}$, Φ, V)
6	(X*i*,*j*, QS, V) → (Z*i*, JS, V)
7	(Y*i*, QS, V) → (Z*i*, JS, V)
8	(X*i*,*j*, QS, V) → (X*i*,*j*, KH, V)
9	(X*i*,*j*, QS, V) → (X*i*,*j*, KD, V)
	- - -
	(X*i*,*j*, 6H, V) → (Z*i*, 5H, V)
	(Y*i*, 6H, V) → (Z*i*, 5H, V)
	(X*i*,*j*, 6H, V) → (X*i*,*j*, 7S, V)
	(X*i*,*j*, 6H, V) → (X*i*,*j*, 7C, V)
	- - -
46	(X*i*,*j*, 2C, V) → (Z*i*, AC, V)
47	(Y*i*, 2C, V) → (Z*i*, AC, V)
48	(X*i*,*j*, 2C, V) → (X*i*,*j*, 3H, V)
49	(X*i*,*j*, 2C, V) → (X*i*,*j*, 3D, V)

Table1: Allowable Transitions

We are hopeful that generalizable patterns will be discovered by working with a little version of the game, and that, together with any improvements in the state descriptions, a road to solving this problem may present itself.